Non-Gaussian Resistance Noise near Electrical Breakdown in Granular Materials

C. Pennetta [a,*], E. Alfinito [a], L. Reggiani, [a] S. Ruffo [b]

[a] *INFM, National Nanotechnology Lab., Via Arnesano, 73100, Italy and Dip. di Ingegneria dell'Innovazione, Università di Lecce, Via Arnesano, 73100, Italy.*

[b] *CSDC and Dipartimento di Energetica "Sergio Stecco", Università di Firenze, Via S. Marta, 3, INFN and INFM, Firenze, 50139, Italy.*

Abstract

The distribution of resistance fluctuations of conducting thin films with granular structure near electrical breakdown is studied by numerical simulations. The film is modeled as a resistor network in a steady state determined by the competition between two biased processes, breaking and recovery. Systems of different sizes and with different levels of internal disorder are considered. Sharp deviations from a Gaussian distribution are found near breakdown and the effect increases with the degree of internal disorder. However, we show that in general this non-Gaussianity is related to the finite size of the system and vanishes in the large size limit. Nevertheless, near the critical point of the conductor-insulator transition, deviations from Gaussianity persist when the size is increased and the distribution of resistance fluctuations is well fitted by the universal Bramwell-Holdsworth-Pinton distribution.

Key words:
Non-Gaussian distributions, Nonequilibrium steady states, Electrical breakdown
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1 Introduction and Model

Non-Gaussian distributions of several quantities characterizing the behavior of complex systems in non-equilibrium steady-states, have been evidenced since long time in many experiments [1]. The general properties of these anomalous distributions and their link with other properties of non-equilibrium complex

* Corresponding author. Dipartimento di Ingegneria dell'Innovazione, Università di Lecce, Via Arnesano, 73100, Italy.
Email address: `cecilia.pennetta@unile.it` (C. Pennetta).
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systems is far from being fully understood. Rather, the study of these distributions is nowadays attracting an increasing interest in the literature [2–7]. Here, we study the distribution of resistance fluctuations of a conducting thin film with granular structure near electrical breakdown. Electrical breakdown phenomena typically occur in conductors stressed by high current densities and they consist in an irreversible and dramatic increase of resistivity of the material and thus they are associated with a conductor-insulator transition [8–12]. In our study we make use of the Stationary Network Under Biased Percolation (SNUBP) model [13]. This model provides a good description of many features associated with the electrical instability of composites materials [11] and with the electromigration damage of metal lines [14], two important classes of breakdown phenomena. The film is modeled as a resistor network which reaches a steady state determined by the competition between two biased stochastic processes, breaking and recovery. Systems of different sizes and with different levels of internal disorder are considered, where the disorder is mainly related to the fraction of broken resistors within the network. The resistance and its fluctuations are calculated by Monte Carlo simulations which are performed under different stress conditions. Resistance fluctuations are found to deviate from Gaussianity near electrical breakdown. We analyze and discuss this non-Gaussianity in the spirit of the Bramwell-Holdsworth-Pinton (BHP) distribution, recently introduced in the context of the study of highly correlated systems near criticality [3,4]. In particular, we show that the deviations from Gaussianity observed near the breakdown are related to finite size effects, and consequently vanish in the thermodynamic limit. Nevertheless, near the percolation critical point the non-Gaussianity persists in the large size limit and it is described by the universal BHP distribution.

According to the SNUBP model[13], a conducting film with granular structure is described as a two-dimensional resistor network. Precisely, we consider a square-lattice of $N \times N$ resistors. This lattice lies on an insulating substrate at a given temperature T_0, which acts as a thermal bath. Each resistor can be in two different states: (i) regular, corresponding to a resistance $r_n = r_0[1 + \alpha(T_n - T_0)]$ and (ii) broken, corresponding to the effectively "infinite" resistance, $r_{OP} = 10^9 r_n$ (in the following we call this state defect). Here, α is the temperature coefficient of the resistance and T_n the local temperature. This latter is determined by taking into account Joule heating effects and thermal exchanges between neighbor resistors [12]: $T_n = T_0 + A[r_n i_n^2 + (3/4N_{neig}) \sum(r_l i_l^2 - r_n i_n^2)]$, where i_n is the current flowing in the nth resistor, N_{neig} the number of nearest neighbors over which the summation is performed. The parameter A represents the thermal resistance of each resistor and determines the importance of Joule heating effects. By taking the above expression for T_n we are neglecting time dependent effects in heat diffusion studied by Sornette et al. [9]. A constant stress current I is then applied through perfectly conducting bars at the left and right sides of the network. The two biased processes consist of stochastic transitions between the two possible states of each resistor and they occur, through thermal activation, with probabilities: $W_{Dn} = exp[-E_D/k_BT_n]$ and $W_{Rn} = exp[-E_R/k_BT_n]$, characterized by the two activation energies, E_D and E_R [13,15] (k_B is the Boltzmann constant). The network time evolution is obtained by a Monte Carlo simulation which updates the network resistance after breaking and recovery processes, according to the iterative procedure described in details in Ref. [13]. The sequence of successive configurations provides a resistance $R(t)$ signal after an appropriate calibration of the time scale. Then, depending on the stress conditions (I and T_0) and on the network parameters (sizes, activation energies and other parameters related to the material like r_0 and α), the network reaches a steady state or undergoes an irreversible electrical failure. This last possibility is associated with the achievement of the percolation threshold, p_c, for the fraction of broken resistors. Therefore, for a given network at a given temperature, a threshold current value, I_B, exists above which electrical breakdown occurs [13]. Below this threshold, the steady state of the network is characterized by fluctuations of the fraction of broken resistors, δp, and of the resistance, δR, around their respective average values $< p >$ and $< R >$. In particular, we underline that the ratio $(E_D - E_R)/k_BT_0$ determines the average fraction of defects for a given value of $I < I_B$ and thus the level of disorder inside the network. In the following section we analyze the results of simulations carried out by considering networks of different sizes, with different levels of disorder and stressed by different currents. In all cases we take $T_0 = 300$ (K), $E_D = 0.170$ (eV), $r_0 = 1$ (Ω), $\alpha = 10^{-3}$ (K^{-1}), $A = 5 \times 10^5$ (K/W) (these values are chosen as physically plausible). N ranges between $50 \div 125$, while E_R between $0.0259 \div 0.164$ (eV).

2 Results

We report in Fig. 1 the resistance of a 75×75 network as a function of time and at increasing currents. The evolutions $R(t)$ in Fig. 1 are obtained by taking $E_R = 0.103$ (eV) for the activation energy of the recovery process. This value of E_R provides a network with an intermediate level of disorder. This figure displays two important features of the electrical response of a conducting film: i) the linear regime at low currents is followed by a nonlinear regime where the average resistance increases significantly with the current; ii) the amplitude of the resistance fluctuations increases strongly close to the breakdown. Precisely, the lowest curve in Fig. 1 corresponds to the linear regime, i.e. it is obtained for $I = 0.001(A) < I_0$ where I_0 is the current value associated with the onset of the nonlinearity of the I-V characteristic [13]. The second curve corresponds to the nonlinear regime, $I = 0.45(A) > I_0$, the third to the threshold for the breakdown, $I = I_B = 0.70(A)$, and finally the highest curve to a network undergoing an electrical breakdown at a current $I = 0.75(A) > I_B$. This last curve has been shifted upwards by $0.1(\Omega)$ for graphical reasons. A detailed analysis of the behavior of the average resistance and of the relative variance of resistance fluctuations as a function of the current can be found in Ref. [13].

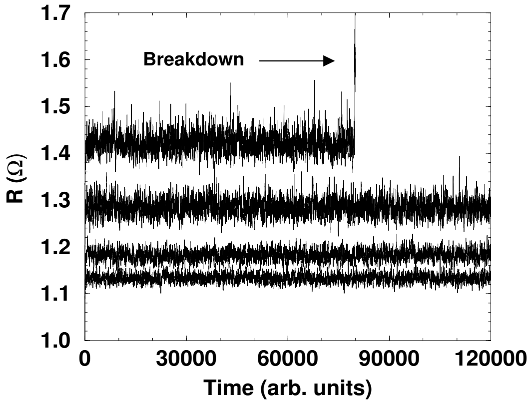

Fig. 1. Resistance evolutions at increasing currents. Starting from the bottom: $I = 0.001$ (A) (linear regime), $I = 0.45$ A) (nonlinear regime), $I = 0.70$ (A) (threshold current), $I = 0.75$ (A) (breakdown). The highest curve has been shifted upwards by $0.1(\Omega)$ for graphical reasons.

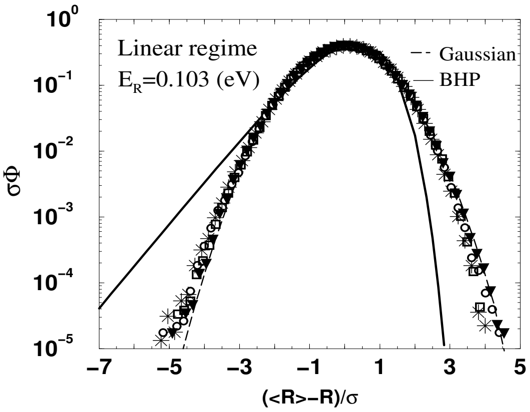

Fig. 2. Normalized PDF of resistance fluctuations in the linear regime at increasing network sizes: 50×50 (stars), 75×75 (small circles), 100×100 (squares), 125×125 (down triangles). The recovery energy is $E_R = 0.103$ (eV). The thick solid curve and the dashed curve correspond to the BHP and Gaussian distributions, respectively.

We have investigated the Gaussianity of the steady state signals shown in Fig. 1. The results of the analysis are reported in Fig. 2 for $I = 0.001$ (A) (linear regime) and in Fig. 3 for $I = I_B = 0.70$ (A) (threshold current for breakdown). Precisely, in these figures we show on a lin-log plot the product $\sigma\Phi$ as a function of $(< R > -R)/\sigma$, where Φ is the probability density function (PDF) of the distribution of δR and σ is the root mean square deviation from the average. This normalized representation, by making the distribution independent of its first and second moments, is particularly convenient to explore the functional form of the distribution. These results, together with all the others presented in this paper have been obtained by considering time series containing about 1.2×10^6 resistance values. Moreover, for comparison, we have also reported the Gaussian distribution (which in this normalized representation has zero mean and unit variance) and the Bramwell-Holdsworth-Pinton (BHP) distribution

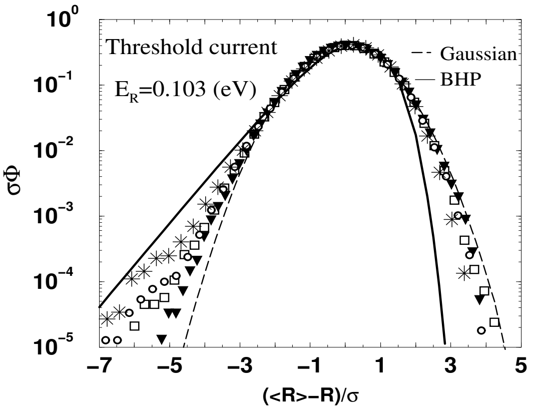

Fig. 3. Normalized PDF of resistance fluctuations at the threshold current, I_B, at increasing network sizes: 50×50 (stars), 75×75 (small circles), 100×100 (squares), 125×125 (down triangles). The recovery energy E_R is the same of Fig. 2. The thick solid and the dashed curves also have the same meaning.

[3,4]. These authors have proposed a universal non-Gaussian PDF for the fluctuations of global quantities of systems at criticality[3]. Denoting m a fluctuating quantity (for example the magnetization of a ferromagnet like in Ref. [4]), $< m >$ and σ_m its mean value and root mean square deviation respectively, $P(m)$ its PDF, $y \equiv (m- < m >)/\sigma_m$ the normalized variable, $\Pi(y) \equiv \sigma_m P(y)$ the normalized PDF and $x \equiv b(y-s)$, the BHP distribution takes the following expression [4]:

$$\Pi(y) = K[e^{x-e^x}]^a \tag{1}$$

where $a = \pi/2$, $b = 0.936 \pm 0.002$, $s = 0.374 \pm 0.001$ and $K = 2.15 \pm 0.01$. This expression can be considered as a generalization of the Gumbel distribution, which is often associated with the occurrence of rare events. In Figs. 2 and 3 together with the PDF obtained for networks with linear sizes $N = 75$ (small circles) are also displayed the PDF corresponding to $N = 50$ (stars), $N = 100$ (squares) and $N = 125$ (down triangles). Figure 2 shows that in the linear regime the distribution of δR is Gaussian for all system sizes. Near breakdown, at $I = I_B$, (Fig. 3) the PDF shows non-Gaussian tails. However, this non-Gaussianity progressively vanishes when the system size is increased. Therefore, when networks with intermediate level of disorder are considered, as in the case of Figs. 2 and 3, deviations from Gaussianity are related to the finite size of the system. The PDF in this case is well fitted by the Eq. (1) once the parameters a, b, s and K are considered as fitting parameters, as described in Ref. [16]. Such phenomenological fits can provide helpful tools in the study of failure precursors in the case of finite size systems.

We have also considered the role of disorder in the breakdown process and its effect on the distribution of resistance fluctuations. At given values of E_D and T_0, the parameter controlling the disorder inside the network, i.e. the average fraction of broken resistors $< p >$, is the activation energy E_R. Figure 4 shows the average fraction of broken resistors versus the external current for different values of E_R. The data are obtained for a 75×75 network. The different sets of

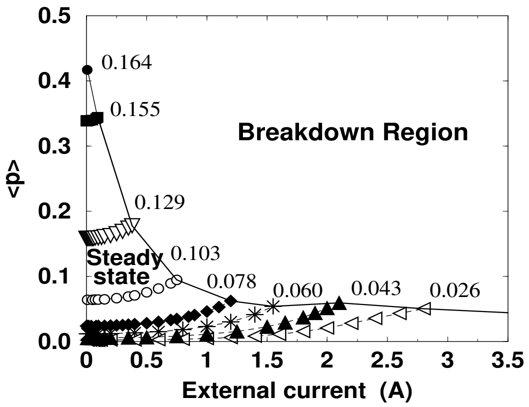

Fig. 4. Average fraction of defects versus current in steady-state networks for different values of the recovery energy E_R. The numbers at the top of each curves indicate E_R in eV. In the region above the solid curve the system breaks down.

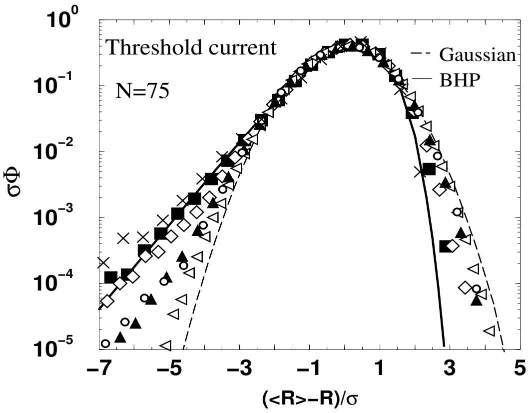

Fig. 5. Normalized PDF of resistance fluctuations of a 75×75 network at increasing E_R values. In all cases the current corresponds to the threshold value. Precisely, left triangles: $E_R = 0.026$, $I_B = 2.6$; full triangles: $E_R = 0.043$, $I_B = 2.1$; small circles: $E_R = 0.103$, $I_B = 0.70$; diamonds: $E_R = 0.140$, $I_B = 0.25$; full squares: $E_R = 0.155$, $I_B = 0.085$, crosses: $E_R = 0.164$, $I_B = 0.0065$ (energies in eV, currents in A).

symbols refer to steady states of the network. The region above the solid curve corresponds to electrical breakdown (in this region the stable states of the network are non-conducting states). Thus, Fig. 4 represents the phase diagram of the system in the $< p > -I$ plane. It can be shown [17] that all the curves in Fig. 4 collapse onto a single one after normalization of I to I_0 and of $< p >$ to $< p >_0$ (the defect fraction in the limit of vanishing current). Moreover, the relative variation of the defect fraction, $[< p > - < p >_0]/ < p >_0$, scales as the ratio I/I_0 with a quadratic exponent [13]. Furthermore, we note that the electrical breakdown corresponding to the values of E_R considered in Fig. 4, is associated with a first order phase transition [17]. These results agree with the behavior observed in electrical breakdown experiments, performed in the Joule regime of composites [11]. Nevertheless, it can be shown [17] that when E_R reaches its maximum value, $E_{R,MAX}$, (determined by the stability condition obtained for a system of given size in the vanishing current limit [15,17]),

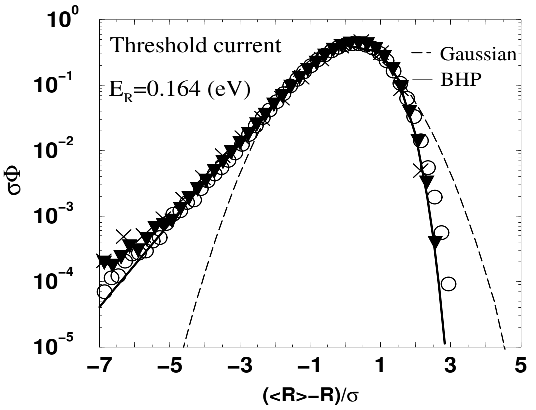

Fig. 6. Normalized PDF of resistance fluctuations at the threshold current, I_B, at increasing network sizes: 75×75 (crosses), 100×100 (down triangles), 125×125 (full circles). The recovery energy is $E_R = 0.164$ (eV).

the conductor-insulator transition becomes of the second order. This change in the nature of the transition, when going from small to high disordered systems, also agrees with the predictions of Andersen et al. [9]. Therefore, we have reported in Fig. 5 the normalized PDF calculated at the breakdown threshold for different values of E_R. Again, a 75×75 network is considered. We can see that the non-Gaussianity of the distribution at $I = I_B$ increases significantly when increasing the E_R value. However, as already shown in Fig. 3, this non-Gaussianity vanishes in the large size limit. Nevertheless, when the value of E_R is nearly equal to $E_{R,MAX}$, the system approaches the critical point and the PDF tends to achieve the BHP form, independently of system size. This is shown in Fig. 6 which displays the normalized PDF calculated at the breakdown threshold, $I = I_B$, for $E_R = 0.164$ (eV). The three sets of data are obtained for $N = 75$, $I = 0.0065$ (A) (crosses), $N = 100$ and $I = 0.009$ (A) (down triangles) and $N = 125$ and $I = 0.011$ (A) (full circles) and correspond to networks near their critical point (the data for $N = 50$ are not shown in Fig. 6 because 50×50 networks are always unstable for this value of E_R). We can see that in this case the PDF becomes independent of the sizes of the system and it is well fitted by the BHP distribution.

3 Conclusions

We have studied the distribution of the resistance fluctuations of conducting thin films with granular structure near electrical breakdown. The study has been performed by describing the film as a resistor network and by using the SNUBP model [13]. We have considered systems of different sizes and with different levels of internal disorder. A non-Gaussianity of the fluctuation distribution is found near electrical breakdown. This non-Gaussianity increases with the degree of disorder of the network. However, we have shown that this non-Gaussianity is related to the finite size of the system and that it vanishes in the large size limit. Nevertheless, near the critical point the non-Gaussianity persists in the large size limit and is well fitted by the universal Bramwell-Holdsworth-Pinton distribution [3–5].

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